1.7 - Linear Functions

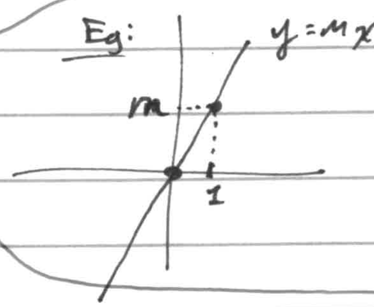
Define: $f(x)$ is a linear function if it is defined by

$$y = f(x) = mx + b$$

m: slope (up 1, over m)

b: y-intercept

Eg:



Lines are easy to work with.

We can find a line from

(I) a point (x_1, y_1) & a slope m

$$\boxed{y = f(x) = m(x - x_1) + y_1}$$ Point slope form

(II) two points (x_1, y_1) and (x_2, y_2)

step 1: slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1}$

step 2: $f(x) = m(x - x_1) + y_1$

skip during class

Eg: find the line through $(\underset{x_1}{2}, \underset{y_1}{3})$ and $(3, 5$

step 1: $m = \dfrac{5-3}{3-2} = \dfrac{2}{1} = 2$

step 2: $f(x) = 2(x - 2) + 3$

(III) a y-intercept & another point

eg y-intercept = 2

& goes through (1, 1)

step 0: Rember y-intercept = $f(0)$

$\Rightarrow$ (0, 2) is a 2nd point on the line

Same as before

step 1: $m = \dfrac{2-1}{0-1} = \dfrac{1}{-1} = -1$

step 2: $f(x) = m(x - x_1) + y_1$

(5)

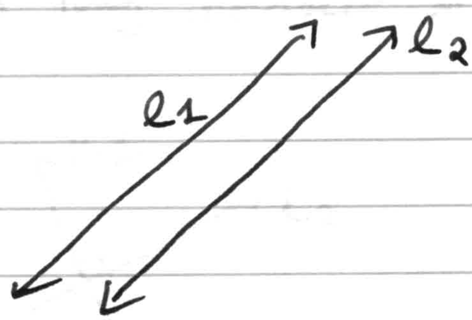
skip during class

$f(x) = -1(x - \bigcirc) + \bigcirc$

your choice: x-value y-value

__Define:__ two lines are __parallel__ if they never touch

__Eg:__



5 The slope of the 2^{nd} = the slope of the 1^{st}

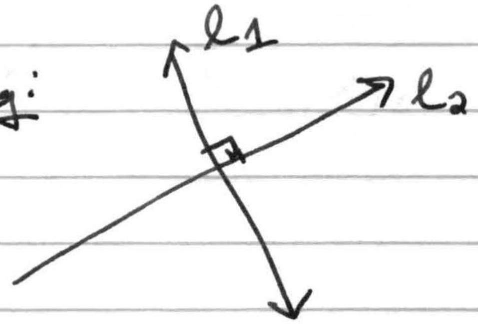
So given a line l_1 and a point not on the line P

we can make a line parallel to l_1 going through P
By using point-slope form

<u>Define:</u> two lines are <u>perpendicular</u>

if they intersect at right angles

<u>Eg:</u>



5

The slope of the 2^{nd} is $\dfrac{-1}{\left(\text{the slope of the first}\right)}$

ℓ_1 has slope $m \iff \ell_2$ has slope $\frac{-1}{m}$

$\Rightarrow$ you can find perpindicular lines using only a line (slope) and another (point)

[sleep during class]

Eg: let l_1 be $y = 3x + 1$

& P be $(1, 2)$

Find $f(x)$ parallel to l_1 & going through P

$$f(x) = m(x - x_1) + y_1$$

step 1: f is parallel to l_1

$\Rightarrow$ they have the same slope

$\Rightarrow m = 3$

5-10

step 2: $f(x) = 3(x - 1) + 3$

Next find $g(x)$ perpendicular to l_1 & going through

$$g(x) = m(x - x_1) + y$$

step 1: g is perpendicular to l_1

$\Rightarrow m \cdot 3 = -1$

$\Rightarrow m = \frac{-1}{3}$

step 2: $f(x) = \frac{-1}{3}(x - 1) + 3$

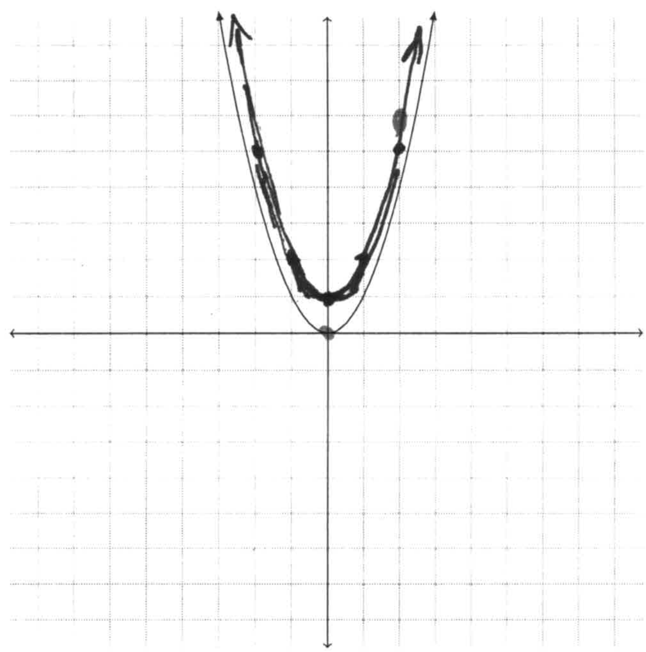
Graphing Functions Worksheet Math 1060 September 10, 2013

Name: ______________________________ Section: ______________________________

The graph of $y = x^2$ is given.

$\boxed{\text{for each } x \text{ in domain plot } (x, f(x))}$

Graph $f(x) = x^2 + 1$



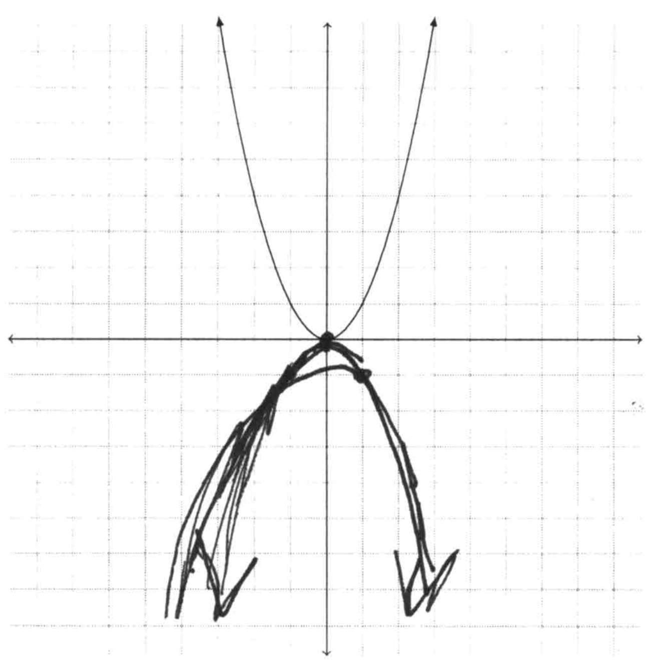
x	$f(x)$
-1	$1+1 = 2$
0	1
1	$1+1 = 2$
2	$4+1 = 5$

Domain: $(-\infty, \infty)$

Range: $[1, \infty)$

This is x^2 moved up 1

Graph $f(x) = -x^2$



x	$f(x)$
-1	$-(-1)^2 = -1$
0	0
1	$-(1)^2 = -1$

Domain: $(-\infty, \infty)$

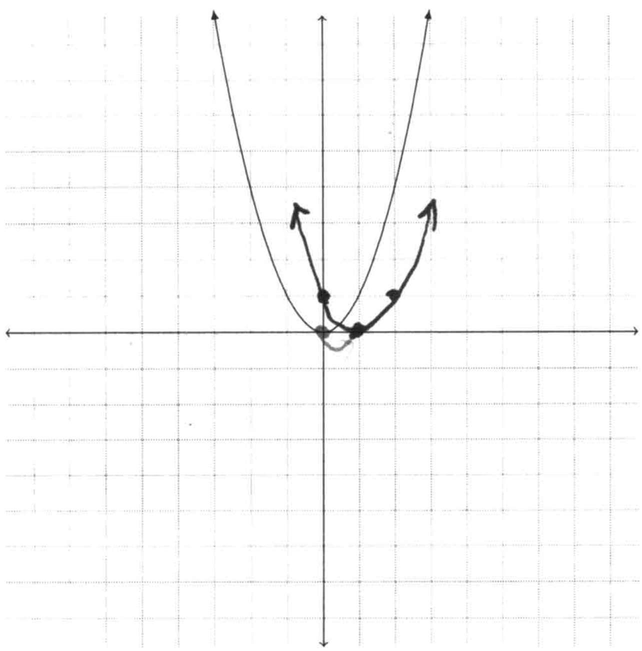
Range: $(-\infty, 0]$

Graphing Functions Worksheet Math 1060 September 10, 2013

Name: ______________________ Section: ______________________

The graph of $y = x^2$ is given.

Graph $f(x) = (x-1)^2$



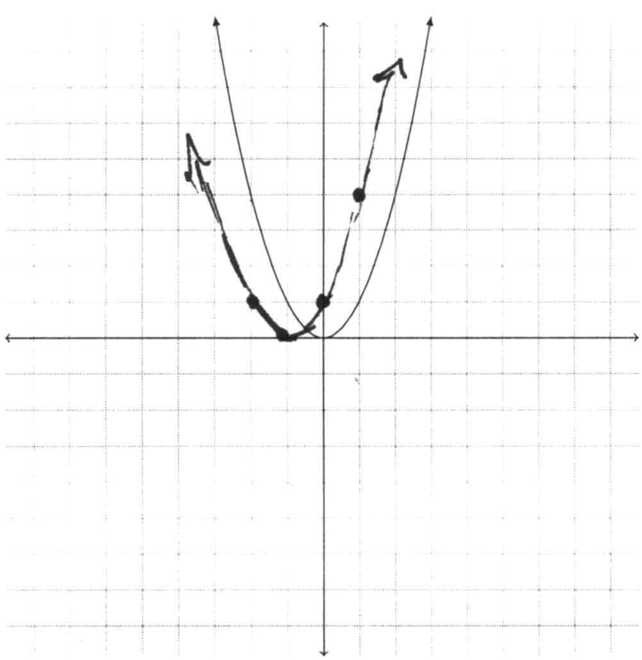
x	$f(x)$
0	$(0-1)^2=1$
1	$(1-1)^2=0^2=0$
2	$(2-1)^2=1$

Domain: $(-\infty,\infty)$

Range: $[0,\infty)$

x^2 moved Right one

Graph $f(x) = \cancel{x^2}$ $(x+1)^2$



x	$f(x)$
-2	$(-2+1)^2=(-1)^2=1$
-1	$(-1+1)^2=0$
0	$(0+1)^2=1$
1	$(1+1)^2=4$

Domain:

Range:

Say ONLY

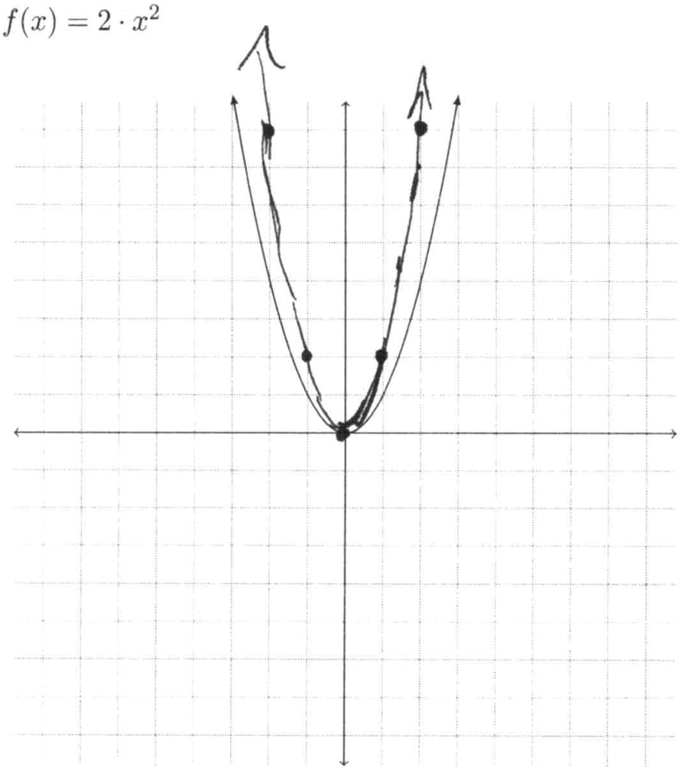
write

x^2 moved left one

Name: ______________________ Section: ______________________

The graph of $y = x^2$ is given.

Graph $f(x) = 2 \cdot x^2$



x	$f(x)$
-1	$2(-1)^2 = 2$
0	$2 \cdot 0^2 = 0$
1	$2 \cdot (1)^2 = 2$
2	$2(2)^2 = 8$

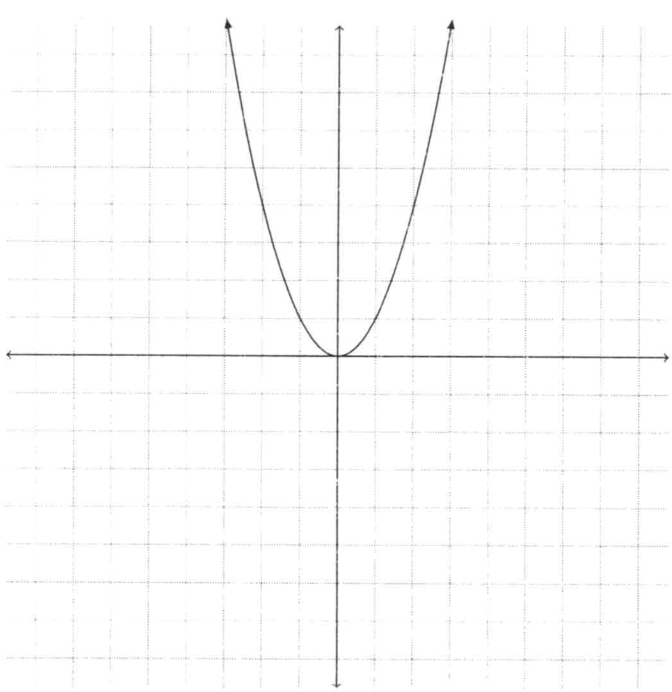
Domain:

Range:

x^2 stretched vertically

Graph $f(x) = \frac{1}{3} \cdot x^2$

Say only



x	$f(x)$

Domain:

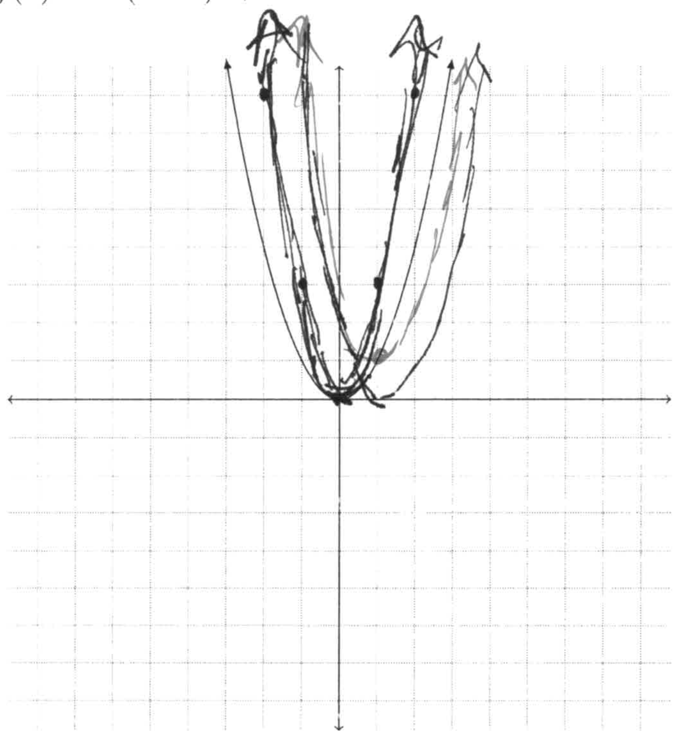
Range: write

x^2 squashed vertically

Name: ______________________ Section: ______________________

The graph of $y = x^2$ is given.

Graph $f(x) = 2 \cdot (x-1)^2 + 1$



Helper Graphs

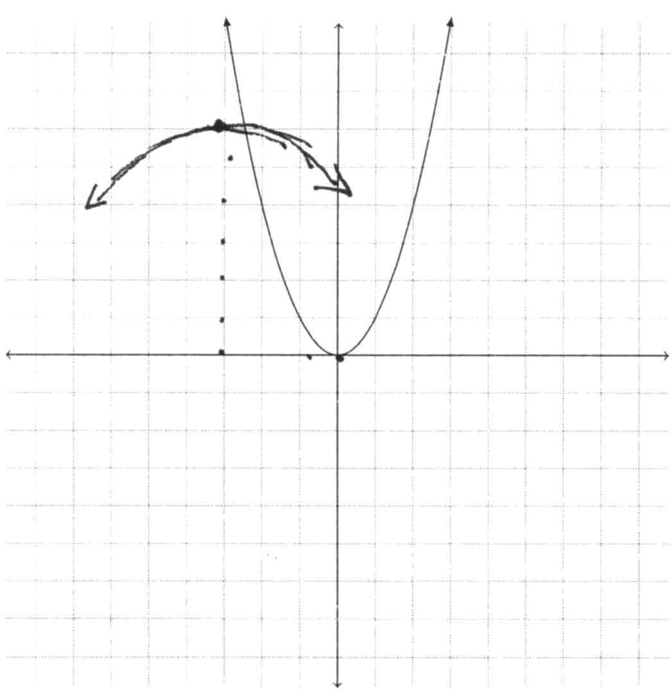
1. $2x^2$
2. $2(x-1)^2$
3. $2(x-1)^2 + 1$

Domain:

Range:

$2x^2$, moved right one then up one

Graph $f(x) = -\frac{1}{3} \cdot (x+3)^2 + 6$



Helper Graphs

1.

2.

3.

4.

Domain:

Range:

$\frac{1}{3}x^2$, flipped across x-axis, moved left 3, & moved ~~3~~ up 6

Wouldn't it be nice

if <u>every</u> quadratic function

had the form

$$f(x) = a(x+k)+h$$

some #'s

Then we could graph it quickly!

$y = ax^2$ moved left/right k

up/down h

<u>Cool Fact</u>: we can always do this!

<u>Eg</u>: we can turn

$$h(x) = x^2 - 4x + 3$$

into

$$h(x) = (x-2)^2 - 1$$

The idea is to complete the square:

Old way:

$$h(x) = x^2 - 4x + 3$$

$$h(x) - 3 = x^2 - 4x$$

$(-2)(-2) = 4$

$$h(x) - 3 + \underline{4} = x^2 - 4x + \underline{4}$$

$\frac{-4}{2} = -2$

$$h(x) - 3 + \underline{4} = (x-2)(x-2)$$

$$h(x) + 1 = (x-2)^2$$

$$h(x) = (x-2)^2 - 1$$

New way:

$$h(x) = x^2 - 4x + 3$$

$(-2)(-2)$

$$= \left[x^2 - 4x + \underline{4}\right] - \underline{4} + 3$$

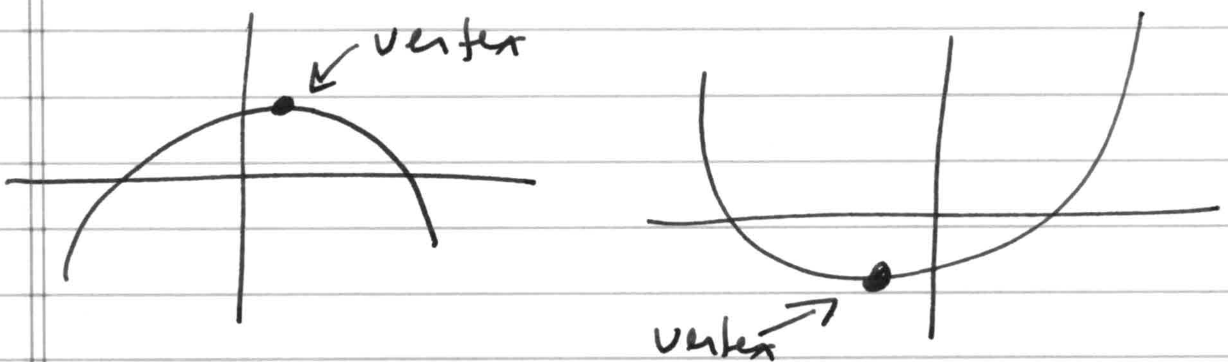
$\frac{-4}{2} = -2$

$$= \left[(x-2)(x-2)\right] - 4 + 3$$

$$h(x) = (x-2)^2 - 1$$

The *vertex* of a parabola

is the bottom (or top) of the cup



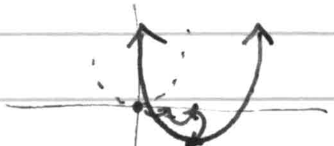
The vertex of $h(x) = a(x+k)+h$

is $(-k, h)$

where the squared term is zero

how far up/down the bottom moved

Eg: $h(x) = (x-2)^2 - 1$ is x^2 moved RIGHT 2 and DOWN 1



$\boxed{\text{vertex of } h(x) \text{ is } (2,-1)}$